Piezoelectric-Crystal-Resonator High-Frequency Gravitational Wave Generation and Synchro-Resonance Detection

Robert M. L. Baker, Jr. 1, R. Clive Woods2, and Fangyu Li3

1GRAVWAVE LLC, 8123 Tuscany Avenue, Playa del Rey, CA 90293, USA
2Department of Electrical and Computer Engineering and Microelectronics Research Center, 2128 Coover Hall, Iowa State University, Ames, Iowa 50011–3060, USA.
3Department of Physics, Chongqing University, Chongqing 400044, P.R. CHINA.
1(310) 823-4143, robert.baker.jr@comcast.net

Abstract. Here we show the generation of high-frequency-gravitational-waves (HFGWs) utilizing piezoelectric elements such as the ubiquitous Film-Bulk-Acoustic-Resonators (FBARs), found in cell phones, as energized by inexpensive magnetrons, found in microwave ovens, generating GWs having a frequency of about 4.9GHz and their detection by means of new synchro-resonance techniques developed in China. In the 1960s Weber suggested piezoelectric crystals for gravitational-wave (GW) generation. Since then researchers have proposed specific designs. The major obstacle has been the cost of procuring, installing, and energizing a sufficient number of such resonators to generate sufficiently powerful GWs to allow for detection. Recent mass-production techniques, spurred on by the production of cell phones, have driven the cost of resonators down. The new Chinese detector for detecting the 4.9x10^9Hz HFGW is a coupling-system of fractal membranes-beam-splitters and a narrow, 6.1cm-radius, pulsed-Gaussian-laser or continuous-Gaussian detection beam passing through a static 15T-magnetic field. The detector is sensitive to GW amplitudes of ~10^{-30} to be generated with signal-to-noise ratios greater than one. It is concluded that a cost-effective HFGW generation and detection apparatus can now be fabricated and operated in the laboratory. If the two groups or clusters of magnetrons and FBARs were space borne and at lunar distance (e.g., at the Moon and at the lunar L3 libration point) and the quadrupole formalism approximately holds for GW radiators (the FBAR clusters) many GW wavelengths apart, then the HFGW power would be about 420 W and the flux about 2x10^5 Wm^{-2} (or more than one hundred times greater than the solar radiation flux at the Earth) focused at the focal spot, or remote-HFGW-emitter, anywhere in the Earth’s environs – on or below the Earth’s surface.

Keywords: Microwaves, Gravitational Waves.
PACS: 04.30Db, 04.80.Nn, 84.40.Fe, and 77.65.-j.

INTRODUCTION

According to accepted definitions (Hawking and Israel, 1979) high-frequency gravitational waves (HFGWs) have frequencies in excess of 100kHz (pulses less than 10μs duration) and may have the most promise for terrestrial generation and practical, scientific, and commercial application. The roots of HFGW research are similar to the roots of low-frequency-gravitational-wave (LFGW) research, which has spawned the LIGO, Virgo, GEO600, and other such projects as well as the proposed LISA experiments – all essentially proposals to detect gravitational waves (GWs). Einstein’s (1916) paper (that first analytically suggested the existence of GWs) and, the research of Joseph Weber (1964), Robert Forward (1966), M.E. Gertsenshtein (1962), L Halpren (1964), and Heinz Dehnen and Romero (1981) commencing in the 1950s, examined the astrophysical generation of LFGW and, especially, the laboratory generation and/or detection of HFGW. Just as the extensive LIGO project has not yet detected LFGWs, there has not yet been an unequivocal demonstration of generating and detecting HFGWs. The present paper is prompted by the increasing likelihood of laboratory-scale generation and detection of HFGW in the near future. This expectation is due to the new availability of inexpensive piezoelectric-resonator-energizable elements and magnetron-energizing elements and the utilization of a new Chinese synchro-resonance detector as discussed, for example, in Li and Baker (2006).
The concept proposed here is to generate and detect HFGWs in the laboratory using a novel arrangement of Film Bulk Acoustic Resonators (FBARs) as discussed by Woods and Baker (2005). They will be energized by phase locked Magnetrons similar to those utilized in ordinary microwave ovens. The details of the derivation of the HFGW generation equations, summarized below, are given by Baker (2000), and the concept of generating HFGWs by an array of micro-devices is described by Baker (2003). From Einstein’s General Theory of Relativity the power of a GW generator, $P$, is approximately given by his quadrupole equation or formalism. This equation for a rotating rod of mass, $m$, can be written as

$$P = \frac{2Gm^2r^4}{6} \frac{\omega^6}{45c^5} = 1.76 \times 10^{-52} \left(2r\Delta f/\Delta t\right)^2 \text{ W},$$  \hspace{0.5cm} (1)

where $r$ is the radius of gyration of a mass (or system of masses, e.g., a rod) in meters (about 300 m for one of the experiments proposed here), $\Delta f$ is an incremental increase in force acting on a mass (e.g., the ends of the rod) in Newtons during an incremental time period $\Delta t$ in seconds; commonly referred to as a “jerk.” For a continuous train of jerks, the frequency is $\nu_{GW} = 1/\Delta t$, and Eq. (1) can be phrased as a function of HFGW frequency as

$$P(r, \Delta f, \nu) = 1.76 \times 10^{-52}(2r \nu_{GW} \Delta f)^2 \text{ W}. \hspace{0.5cm} (2)$$

A method of generating HFGWs is to rotate a rod so that it radiates energy through this quadrupole HFGW generation mechanism. This is impractical because it has long been known that rotation of any material at sufficiently high speed for efficient HFGW generation according to Eq. (1) is impossible; at much lower rotation speeds needed for efficient HFGW generation, centrifugal force causes the rotating mass to disintegrate. This is the origin of the widely-held view that generation of GWs is impossible outside of astronomical bodies. The basic concept proposed here is to utilize piezoelectric mechanical resonators, specifically Film Bulk Acoustic Resonators (FBARs) similar to those utilized in cellular telephones, energized by magnetrons similar to those utilized in microwave ovens, to produce high frequency jerks in the vibrational elements of the FBARs and thereby generate HFGWs. The concept is diagramed in Figs. 1a and 1b, where the rod is dumbbell shaped in Fig. 1a and the change in centrifugal force, $\Delta f_{cf}$, acts on both ends of the rod. In Fig. 1b the two rotating masses are replaced by two clusters of piezoelectric crystals (FBARs). The use of such crystals for HFGW generation is not new. The research of Joseph Weber (1960), of Forward and Miller (1966), of Halpren and Laurent (1964), and of Dehnen (1981), commencing in the 1950s, examined the use of piezoelectric crystals of the laboratory generation of GWs; but none had inexpensive components available for practical designs.

(a)

Two orbiting masses or rotating dumbbell.
Clusters of energizing and energizable elements emulating the HFGW generated by a rotating dumbbell or two orbiting masses.

**FIGURE 1.** Gravitational Wave Generator Geometry

An FBAR (Fig. 2) is a mechanical (acoustic) resonator consisting of a vibrating membrane (typically around $100 \times 100 \mu \text{m}^2$ in plan form, and around $1 \mu \text{m}$ thickness), fabricated using well-established integrated circuit (IC) micro fabrication technology. The vibrating membrane is actuated piezoelectrically, typically using aluminum nitride (AlN) as the piezoelectric excitation material. These devices have recently been highly developed as cost-effective RF filters mainly for use in commercial cellular telephones, in which high-$Q$ resonators are needed for the typical carrier frequency band around 1.9 GHz. Exploratory FBAR devices have been fabricated operating at frequencies up to 7.5GHz (Ruby and Merchant, 1994; Lakin et al., 2001). The vibrating membrane in an FBAR will also form an ideal vibrating mass for the present application, at slightly higher frequency. The FBARs are utilized in cell phones, and thereby cause mechanical deformation or jerking in the crystal-lattice molecules or the vibrating membrane attached to the piezoelectric material of the FBAR and generate GWs. FBARs are a sophisticated development not only of piezoelectric crystals, but also of the quartz crystal resonators found in equipment like electronic watches, computers, TVs, and some radios. FBARs are readily available off the shelf.

**FIGURE 2.** Basic FBAR Construction (Cross-section Side View, Not to Scale).

The FBARS will be arranged in a dumbbell configuration as shown in Fig. 1b, with two groups established at a distance $2r$ apart and each vibrating element will vibrate in phase with all the rest. In this way, an asymmetrical distribution of masses all oscillating at the same frequency will be produced, which reproduces the essentials of a rotating rod except that no centrifugal forces are generated so that the system will not be torn apart by centrifugal forces. All the FBARs jerk in a direction tangential to the line between them and in a common plane. The essential concept here is that a collection of asymmetrical rotating masses, in a dumbbell configuration as shown in, Fig. 1a, with their attendant centrifugal-force jerks, is emulated by two clusters of fixed masses that are jerked by piezoelectric forces – energizable elements (e.g., FBARs) acted upon by energizing elements (e.g., magnetrons), as shown in Fig. 1b. In Fig. 1a, a dumbbell or, alternatively, two masses exhibiting orbital motion (e.g., two stars or black holes) is illustrated by the GW generation by a double-star 1, 2 on orbit 4. Due to the change in the centrifugal-force vectors 8, 9 with time (termed jerks) as the star masses progress around their orbit, GWs are generated 11, 12 and emanate from a focus 3. The GW radiation pattern there is like figure 8 according to Landau and Lifshitz (1975). As depicted in Fig. 3 of Puthoff and Ibison (2003), the radiation-pattern equations of Landau and Lifshitz (1975) give rise to two symmetrical lobes of radiation directed in both directions (thus a figure 8) normal to the plane of the masses motion. Its quantitative value is given by Eq. (10) of Baker, Davis, and Woods (2005). In Fig. 1b this orbital-mass generation process (as previously noted) is emulated by the jerks associated with
piezoelectric crystal energizable elements 16 (e.g., FBARs) as energized by, for example, magnetrons 15. The magnetron-crystal sets are distributed in two compact volumes or clusters 13. For simplicity of illustration the volume of these sets is shown to be spherical, but the volume containing these distributed energizing and energizable elements could be of almost any form. The distribution of the energizable elements must, however, be circumferentially asymmetrical. Specifically, dumbbell-like distributions can emit GWs whereas ring-like distributions do not. For example, rotating rods emit GWs, but revolving rings and spinning balls do not. Such an effect could be explained analytically, but in order to avoid involved mathematical analyses it is possible to provide an intuitive idea of the effect. Consider the tidal effect of the Moon on the Earth’s oceans with two bulges toward and away from the lunar direction. If one added a second Moon having a true anomaly 90° ahead of the first Moon, then there would be four bulges and these tidal crests would not be relatively as high. If we had an entire ring of coplanar moons orbiting the Earth, that is a circumferentially even distribution, the tides would be even and essentially disappear. Such an effect is analogous to the absence of GWs from a circumferentially even mass distribution of a revolving ring. The concept is to energize these asymmetrically distributed elements in a sequence so as to replicate the distributed molecules under gravitational force in the stellar masses of Fig. 1a (1 and 2). As Woodward (2005) wrote: “There is no difference between a whole bunch of small masses actuated in phase and a single large mass that moves quasi-rigidly.” Such a sequence is achieved by controlling the magnetron energizers to emit in sequence as the microwave crests pass each cluster of piezoelectric crystals to be energized by them as the microwaves move toward the center. A simple timing or phase determining program is executed by the computer control logic system to implement the concept. As an illustrative example we shall utilize the same number of magnetrons and FBARs as in Woods and Baker (2005). In this case the resulting Δf of the FBAR ensemble is 4x10^8 N and the HFGW frequency is \( f_{GW} = 4.9 \text{ GHz} \). We chose \( r = 300 \text{ m} \). From Eq. (2) we compute \( P = 2.4x10^{-10} \text{ W} \). The detection area is taken to be at a distance \( D = 0.06 \text{ m} \) from the HFGW focus midway between the ensembles or clusters of FBARs, so that from Eq. (10) of Baker, Davis, and Woods, the flux there is \( F_{GW} = 1.4x10^{-8} \text{ Wm}^{-2} \). From Eqs. (107.11) and (107.12) of Landau and Lifshitz (1975) and Appendix A, we find:

\[
A = \left(8\pi G F_{GW}/c^3 \omega^2 \right)^{1/2} = 1.28x10^{-18} F_{GW}^{1/2} N_{GW},
\]

where \( G \) is the universal gravitational constant, \( c \) is the speed of light, and \( \omega \) is the “effective rotational rate,” we find that the amplitude of the HFGWs is \( 3x10^{-18} \) meters per meter. Equations (1), (2), and (3) may only be valid for \( 2r < \lambda_{GW} \) as, for example, A. Pais (1982), p. 280 and K. S. Thorne (1987), p. 357 indicate. On the other hand, L.P. Grishchuk (2003) suggested that the requirement that \( 2r < \lambda_{GW} \) may not be a stringent or even a necessary one for the quadrupole approximation to GW power to hold. As K. S. Thorne (1987) states “...the quadrupole formalism typically is accurate to within factors of order 2 even for sources with sizes of (the) order (of) a reduced (GW) wavelength ...” Whether the quadrupole approximation to the power of gravitational wave generation holds or not does not necessarily imply that no GWs are generated by an impulsive force acting on a pair of masses or FBAR clusters or that the power does not increase with the distance, \( 2r \), between them. The quadrupole formalism may still provide order-of-magnitude estimates perhaps augmented by higher-order octupole, hexadecapole, etc. modes of pulsation and possibly reduced at the GW focus by diffraction. It is a problem deserving study in the future. Of course, the clusters of magnetrons and FBARs could be fabricated like barbells less than \( 2r < \lambda_{GW} = 6.1 \text{ cm} \) apart. Either way considerable GW should be generated, but the approximations to \( P \) (from the quadrupole formalism) or \( A \) may not accurately hold especially when the two GW radiators, the FBAR clusters, are many GW wavelengths apart.

**HFGW DETECTOR**

The HFGW detector consists of new-type of fractal membranes (Wen, et al., 2002; Zhou, et al., 2003) and a Gaussian beam passing through a static magnetic field (Li and Baker, 2006; Li and Yang, 2004). In the electromagnetic (EM) detector we use the Gaussian beam of fundamental frequency mode (Yariv, 1975), that is,

\[
y = \frac{y_0}{\sqrt{1+(z/f)^2}} \exp\left(-\frac{r^2}{W^2}\right) \exp\left\{i\left[(k_z z - \omega t) - \tan^{-1}\left(\frac{z}{f} + \frac{k_r r^2}{2R} + \delta\right)\right]\right\}, \tag{4}
\]

where the \( y \) variable vector expresses that background static magnetic field \( B \) is pointed along the \( y \)-axis (this is because according to the Einstein-Maxwell equations in the weak GW field, if the propagating direction of the GW is parallel with the static magnetic field, then the perturbation of the GW to the static magnetic field vanishes; while maximum perturbation will occur when the propagating direction of the GW is perpendicular to the static magnetic field \( B \), thus, for the GW propagating along the \( z \)-axis, the best direction of the field \( B \) will be the \( y \)-axis), \( r^2 = x^2 + z^2 \), and \( \delta \) is a phase delay needed to account for the phase delay due to the finite propagation distance.
\[ y^2, \ k_e = \frac{2\pi \lambda_e}{\omega_e}, f = \pi W_0^{2/3}/\lambda_e, W = W_0[1+(z/f)^2]^{1/2}, R = z + f/\omega_e, y_0 = \text{the amplitude of the electric (or magnetic) field of the Gaussian beam}, \ \text{and } \delta^* \text{ is an arbitrary phase factor. Moreover, a static magnetic field pointing along the y-axis is localized in the region } l_z, \ l_z, \ l_z, \ l_z, \ \text{i.e.,} \]

\[
B = \begin{cases} \ 0 & (z \leq -l_z \ \text{and } \ z > l_z), \\ \ B & (l_z \leq z \leq l_z) \end{cases}
\]

as we have shown (Li and Baker, 2006) that under the synchro-resonance condition (when the frequency \( \nu_{GW} \) of the Gaussian Beam is tuned to the \( \nu_{GW} \) of the HFGW), resonant response of the EM detector to the HFGW will be generated, then the perturbative photon flux (PPF) density propagating along the x-axis (notice that the propagating direction of the PPF is perpendicular to the symmetrical axis of the Gaussian beam) will be approximately (Li and Baker, 2006; Li and Yang, 2004) given by:

\[
n_s^{(i)} \approx \frac{1}{\mu_e \hbar \omega_e} AB y_0. \tag{6}
\]

Thus the total PPF passing through the effective receiving surface (the surface area, \( \delta^* s \), is approximately the area of the Gaussian beam’s cross-section having a radius of one GW wavelength of 6.1 cm, so that \( \delta^* s = \pi(0.061)^2 \approx 1.13 \times 10^{-2} \text{m}^2 \)) will be:

\[
N_s^{(i)} \approx n_s^{(i)} \delta^* s = \frac{1}{\mu_e \hbar \omega_e} AB y_0 \delta^* s. \tag{7}
\]

In the following we shall give some results in different cases. The Gaussian beam is a ultra-high-intensity pulse. If the instantaneous power \( P = 10^{14} \text{W} \) and the spot radius \( W_0 = 0.06 \text{m} \), then the amplitude of electric field of the Gaussian beam would be \( \psi_0 = 3.389 \times 10^9 \text{Vm}^{-1} \). From the above parameters and Eqs. (2), (3) and (7), we obtain the instantaneous values of the PPFs and the detection photon numbers as listed in Table 1.

**TABLE 1.** Perturbative Photons of a Pulsed Gaussian Beam at the Focus for Various HFGW-Generator Radii of Gyration.

<table>
<thead>
<tr>
<th>( r ) (m)</th>
<th>( A )</th>
<th>( V_{GB} = V_{GW} ) (Hz)</th>
<th>( \delta t ), detection duration</th>
<th>( N_s^{(i)} ) (s(^{-1}))</th>
<th>( N_s^{(i)} = N_s^{(i)} \delta t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laboratory size</td>
<td>( 3 \times 10^2 )</td>
<td>( 3 \times 10^{-32} )</td>
<td>( 4.9 \times 10^9 )</td>
<td>( 4.9 \times 10^9 \times 10^{-12} )</td>
<td>( 1.81 \times 10^6 \times 10^{-12} )</td>
</tr>
<tr>
<td>30 km apart</td>
<td>( 3 \times 10^4 )</td>
<td>( 3 \times 10^{-30} )</td>
<td>( 4.9 \times 10^9 )</td>
<td>( 4.9 \times 10^9 \times 10^{-12} )</td>
<td>( 1.81 \times 10^8 \times 10^{-12} )</td>
</tr>
<tr>
<td>Lunar distance</td>
<td>( 4 \times 10^8 )</td>
<td>( 4 \times 10^{-25} )</td>
<td>( 4.9 \times 10^9 )</td>
<td>( 4.9 \times 10^9 \times 10^{-12} )</td>
<td>( 6.31 \times 10^{12} \times 10^{-12} )</td>
</tr>
</tbody>
</table>

Table 1 shows that using the Gaussian beam of the ultra-high-intensity pulse, one might obtain large instantaneous PPFs (~ \( 10^6 \) to \( 10^{12} \text{s}^{-1} \)). However, because of very short detection duration, the total detection photon number in the duration will be very small values, and only under the condition of the lunar distance (\( r \approx 4 \times 10^8 \text{m} \)), we have \( N_r = 21.4 \). Thus, the very short detection duration limited the effective detection photon number. Fortunately, here the HFGW generated by the continuous train of jerks will also be a continuous wave train having a frequency of \( \nu_{gw} = 4.9 \text{GHz} \). Therefore, if we using continuous Gaussian beam and not the pulse Gaussian beam to detect it, then the requirement of relevant parameters will be greatly relaxed, even if the instantaneous PPFs will be much less than the values in the Table 1. For example, if the pulse Gaussian beam is replaced by the continuous Gaussian beam with \( W_0 = 6 \text{cm} \) and \( P = 10^4 \text{W} \), then \( \psi_0 = 1.17 \times 10^3 \text{V} \text{m}^{-1} \). According to Eqs. (2), (3) and (7), using the same means, we obtain the relevant results as listed in Table 2.

**TABLE 2.** Perturbative Photons of a Continuous Gaussian Beam at the Focus for Various HFGW-Generator Radii of Gyration.

<table>
<thead>
<tr>
<th>( r ) (m)</th>
<th>( A )</th>
<th>( V_{GB} = V_{GW} ) (Hz)</th>
<th>( N_s^{(i)} ) (s(^{-1}))</th>
<th>( N_r^{(i)} = N_s^{(i)} \delta t )</th>
<th>( \delta t = 10(\text{s}) )</th>
<th>( \delta r = 10^4(\text{s}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laboratory size</td>
<td>( 3 \times 10^2 )</td>
<td>( 3 \times 10^{-32} )</td>
<td>( 4.9 \times 10^9 )</td>
<td>( 4.9 \times 10^9 \times 10^{-12} )</td>
<td>( 6.13 \times 10^8 \times 10^{-12} )</td>
<td>( 6.13 \times 10^8 \times 10^{-12} )</td>
</tr>
<tr>
<td>30 km apart</td>
<td>( 3 \times 10^4 )</td>
<td>( 3 \times 10^{-30} )</td>
<td>( 4.9 \times 10^9 )</td>
<td>( 4.9 \times 10^9 \times 10^{-12} )</td>
<td>( 6.13 \times 10^4 \times 10^{-12} )</td>
<td>( 6.13 \times 10^4 \times 10^{-12} )</td>
</tr>
<tr>
<td>Lunar distance</td>
<td>( 4 \times 10^8 )</td>
<td>( 4 \times 10^{-25} )</td>
<td>( 4.9 \times 10^9 )</td>
<td>( 4.9 \times 10^9 \times 10^{-12} )</td>
<td>( 2.14 \times 10^7 \times 10^{-12} )</td>
<td>( 2.14 \times 10^7 \times 10^{-12} )</td>
</tr>
</tbody>
</table>
Table 2 shows that although the instantaneous PPFs or \( N_x^{(1)} \) in Table 2 are much less than that in Table 1, longer detection duration, \( \delta t \) will generate larger detection photon number \( N_x^{(1)} \) and cause better signal accumulation effect. Consequently, Table 2 provided a more realistic detecting scheme. As for the direct detection of the PPFs, as we have shown (Li and Baker, 2006; Li and Yang, 2004) that utilizing very different physical behavior between the PPFs and the background photon fluxes (BPFs) in some local regions, they can be split by the special fractal membranes (Zhou, et al., 2003; Wen, et al., 2002), so that, the PPF, in principle, would be observable. For example, \( N_x^{(1)} \) (signal) and \( N_x^{(0)} \) (the x-component of the BPF) propagate along the negative and positive directions of the x-axis in the first octant (the region of \( x, y, z > 0 \), see Fig. 3, Li and Baker (2006)), respectively. Using the reflecting fractal membrane with the normal direction parallel to the \( N_x^{(0)} \), the photon flux reflected by the fractal membrane will be \( N_x^{(1)} \) and not \( N_x^{(0)} \). Once \( N_x^{(1)} \) is reflected, \( N_x^{(1)} \) and \( N_x^{(0)} \) will have the same propagating direction. However, after \( N_x^{(1)} \) is reflected, it can keep its strength invariant within one meter distance to the fractal membrane (Wen, et al., 2002), while \( N_x^{(0)} \) decays as \( \exp(-2r^2/W^2) \) (Li and Baker, 2006; Li and Yang, 2004), then the ratio \( N_x^{(1)} / N_x^{(0)} \) (the signal-to-background noise ratio) would be larger than one in the whole region of \( 0.53 \leq x \leq 1 \) (x is the distance to the fractal membrane). In order to suppress the thermal noise, the EM detector should be cooled down to \( kT < h\omega e \) (k is Boltzmann’s constant), which corresponds to \( T < 0.24K \) for the PPF of \( \nu = 4.9GHz \). For the possible external EM noise sources, using a Faraday cage or shielding covers made from such fractal membranes would be very helpful. In this case, we would be to obtain a good signal-to-noise ratio and an almost pure signal photon flux in the special local regions and in a temperature environment of \( T < 0.24K \). Please see Appendix B.

There is a question as to estimating the GW power between the FBAR clusters, i.e., interior to the radiating system where D<2r, i.e., less than the dimensions of the system of FBAR clusters 2r apart. Although GW will be generated, this is a subject that will require additional theoretical study and the perturbative photon flux maybe even larger and the HFGWs easier to detect.

\[ \begin{align*}
\text{FIGURE 3. } N_x^{(0)}, N_x^{(1)} \text{ and } N_x^{(1)}, \text{ in the 1st and 2nd Octants.}
\end{align*} \]

CONCLUSION

According to current theoretical understanding, generation and detection of GW is now feasible on a laboratory scale and undertaking such an experiment will form a strong test of the theory. It is proposed to use synchro-resonance of EM radiation to detect the HFGWs. Continuous Gaussian beam detection duration on the order of 100 to 1000s gives an adequate signal-to-noise ratio for detection and communications (Baker, 2000). Installation of opposing magnetron-FBAR groups or clusters on two satellites on coplanar geosynchronous orbits located on opposite sides of the Earth, together with careful control of their elements relative phasing and orientation, will enable positioning of the HFGW focus or HFGW emission point at any location between the two satellites around the Earth. One would have a huge GW space antenna – this one for transmitting analogous to the Laser Interferometer Space Antenna or LISA for receiving GWs. In this case a slight timing (phase) or orientation adjustment of the two clusters would allow one to locate the GW focus, which would be a few centimeters across
and emit HFGW (in a radiation pattern described in Baker, Davis, and Woods, 2005) anywhere in the neighborhood of the Earth or under the Earth’s surface for the purpose of laboratory detection or as a remote source of HFGW. At 4.9 GHz and energizing power 100kW, we compute a generated power $P \approx 4.2$ W. This space antenna yields a diffraction-limited focused HFGW flux of $\sim 2 \times 10^3$ W m$^{-2}$. Alternatively, if the two groups of energizing/energizable elements were at lunar distance (Moon and the L3 lunar libration point (Baker, 1967)), then the transmitted GW power would be $\sim 420$ W and the flux, focused as a HFGW emitter at any point near the Earth, would be about $2 \times 10^7$ W m$^{-2}$. This is more than one hundred times greater than the solar radiation flux at the Earth.

**ACKNOWLEDGMENTS**

The support from the National Basic Research Foundation of China under Grant No 2003CB716300, the National Nature Science Foundation of China under Grant 10575140, the Nature Science Foundation of Chongqing under Grant No 8562, GRAVWAVE® LLC, and Transportation Sciences Corporation are acknowledged. Fang-Yu Li contributed the analyses and design of the HFGW detector. R. Clive Woods supplied the specifications and force analyses for the FBARs. Robert M. L. Baker, Jr. contributed the concept and overall design of the experiment and its application to space technology.

**APPENDIX A**

From Eq. (107.11), p. 347 of Landau and Lifshitz (1975), and considering the Transverse Traceless Gauge (i.e., TT Gauge), the nonzero quantities of the metric perturbation $h_{ij}$ for the GW propagating along the x-axis will be $h_{23}$ and $h_{22} = -h_{33}$. Then the energy flux of the GW is:

$$F_{gw} = ct\omega^0 = \frac{c^3}{16\pi G}\left[h_{23}^2 + \frac{1}{4}(h_{22} - h_{33})^2\right].$$  \hfill (1a)

Where $h_{ij} = \partial h_{ij}/\partial t$. Notice that Eq. (1a) must contain differentiation of $h_{ij}$ to time (see, e.g., Misner, Thorne and Wheeler, 1973, Eq. (35.27)). Clearly, for the monochromatic wave of frequency $\omega$, which propagates along the x-axis, the general form of $h_{ij}$ is:

$$h_{ij} = A\exp\left[i(\omega t - kx)\right].$$  \hfill (2a)

Thus,

$$\bar{h}_{ij} = i\omega A\exp\left[i(\omega t - kx)\right], \quad \text{and} \quad \left(\bar{h}_{ij}\right)^2 = \omega^2 A^2,$$  \hfill (3a)

where $A$ is the amplitude of GW. Because of $h_{22} = h_{33}$, the Eq. (1a) can be reduced to:

$$F_{gw} = \frac{c^3}{16\pi G}\left[h_{23}^2 + \frac{1}{4}(h_{22})^2\right].$$  \hfill (4a)

In general, we can put $h_{23} = h_{22}$, in this case, from Eqs. (3a) and (4a), we obtain:

$$F_{gw} = \frac{c^3}{8\pi G}\left(h_{22}\right)^2 = \frac{c^3}{8\pi G}\omega^2 A^2.$$  \hfill (5a)

Finally, solving for $A$ one finds:

$$A = \left(\frac{8\pi G F_{gw}}{c^2 \omega^2}\right)^{1/2} \approx 1.28 \times 10^{-18} \frac{F_{gw}}{\nu_{gw}}.$$

This is just Eq. (3) of this paper.
APPENDIX B

In general, for the HFGW detectors, four kinds of noise sources must be considered. They are usual environmental low frequency noises (e.g., mechanical, seismic noise, etc.), external EM noise sources, background noise and thermal noise: (1) Since the resonant frequency in our EM detector is 4.9 GHz, which is much larger than usual mechanical noise frequencies ($\nu \leq 1$ kHz).

Thus the requirements of suppressing such noise can be greatly relaxed. (2) For the possible external EM noise sources, using a Faraday cage or shielding covers made from such fractal membranes would be helpful (notice that our HFGW detector can be fabricated in a small region, e.g., ~ 0.3 to 1 m, thus there is no principle technical difficulty). Once the detector is isolated from the outside world by such means, possible noise sources would be the remained thermal photons and self-background action. (3) In order to distinguish the remained thermal photons and the perturbative photon flux (PPF, i.e., signal photon flux), it is necessary to reduce the frequency of the thermal photons so that such frequency will be less than the frequency of the PPF. In other words the EM detector should be cooled down $kT < h\nu$ ($k$ is Boltzmann’s constant), which corresponds to $T < 0.24K$ for the PPF of $\nu_{GW} = 4.9$ GHz. Thus the crucial parameter for thermal noise is the selected frequency rather than the total background photon number. Namely, in this case, the thermal noise can be effectively suppressed as long as the detector can select the right frequency. (4) According to the Einstein-Maxwell equations of the weak field and Gertsenshtein effect, in our EM detector the PPF and the background photon flux (BPF) propagate along opposite directions in the same local region (e.g., the first octant, i.e., the region of $x, y, z > 0$). Thus using reflective fractal membrane with the normal direction parallel (the fractal membrane can provide nearly total reflection for the photon flux in the GHz band, and photon fluxes reflected by the fractal membranes can keep their strength invariant within the distance of one meter (see, e.g., Wen, et al., 2002)). The photon flux reflected by the fractal membrane will be $N_x^{(1)}$ (the PPF) but not the $N_x^{(0)}$ (BPF). (See Fig. 3). In this case $N_x^{(1)}$ (i.e., the PPF reflected by the fractal membrane) can keep strength of 49 s$^{-1}$ invariant within one meter distance away from the fractal membrane (for the 30 km radius of gyration, see Table 2), while the BPF $N_x^{(0)}$ decays as $\exp(-2r^2/W^2)$ (this is a typical property of the Gaussian background photon fluxes, see, e.g. Yariv (1975)).

Numerical calculation shows that the ratio $N_x^{(1)}/N_x^{(0)}$ (the signal-to-background noise ratio) would be larger than one in the whole region of $0.53 < x < 1$ m. For example, the BPF $N_x^{(0)}$ has maximum $\sim 10^{22}$ s$^{-1}$ at $x = \pm 4$ cm, which is much larger than $N_x^{(1)}$. However, $N_x^{(1)}$ can still keep its strength of 49 s$^{-1}$ at $x = 70$ cm, while where $N_x^{(0)}$ will be reduced to $10^4$, and they have the same order of magnitude at $x \sim 53$ cm. This means that one may get S/N ~ 1 at $x \sim 53$ cm and S/N > 1 in the region $53 < x < 100$ cm and the photon detector would receive an almost pure signal photon flux of 49 s$^{-1}$ at $x \sim 70$ cm.

The number of detection photons expresses observable values of the PPF. In general, it should satisfy three conditions: (1) It should be perturbative photons generated by the GWs (including first- and second-orders perturbations, in our HFGW detector, this is first-order perturbation). (2) It can be split from the background photon fluxes otherwise it will be swamped by the background. Such background photons include unperturbed Gaussian beam’s photons and the thermal photons. (3) The PPF should have an observable level in the terminal receiver.

For such conditions the perturbative electric field is produced by the direct interaction of the HFGW with the static magnetic field. According to the Einstein-Maxwell equations of the weak fields, the $y$-component $E_y^{(0)}$ of the perturbative electric field is approximately $B/\mu_0$ ($A$ is the amplitude of the HFGW, see, e.g., Logi and Mickelson. (1977); Boccaletti (1970;) and Li, Tang and Shi (2003)), the $x$-component of the magnetic field of the background beam is $y_0 k/\omega$ roughly. Thus the $x$-component of the energy flux density (Poynting vector) consisted by such first-order perturbative electric field and background magnetic field will be $B/\mu_0 A c y_0 k/\omega = AB y_0$. Because the energy of each photon is $\hbar \omega$, so the corresponding perturbative photon flux propagating along the $x$-axis will be:

$$n_x^{(1)} = \frac{1}{\mu_0 \hbar \omega} AB y_0 . \quad (1b)$$

This is just Eq. (6) in the paper. As is shown above that in a temperature environment of $T < 0.23$ K and in the region of 53 cm<x<100 cm (x is the distance to the fractal membrane, see Fig. 3), the signal photon flux can be split from the background photon flux and the thermal photons, and we would be to obtain a good signal-to-noise ratio (S/N>1), although the S/N ~ 1 in the region 0<x<53 cm. Thirdly, even if $x = 300$ m (HFGW-generator’s radius of gyration according to the quadrupole formalism), we have $N_x^{(1)} = 0.49$ s$^{-1}$. For the detection duration of $\delta t = 10$ s and $\delta t = 10^2$ s, one finds $N_x = 4.9 \times 10^2$ (total perturbative photon number, i.e., detection photon number, see, Table 2), respectively. Consequently, such signal photons, in principle, would be observable at the level of the single photon avalanche effect.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>amplitude of gravitational wave (meters/meter)</td>
</tr>
<tr>
<td>B</td>
<td>$B_0$ ($-l_2 \leq z \leq l_2$) or $= 0$ ($x \leq -l$, and $z &gt; l_2$)</td>
</tr>
<tr>
<td>c</td>
<td>speed of light, $2.998 \times 10^8$ (ms$^{-1}$)</td>
</tr>
<tr>
<td>D</td>
<td>distance from GW focus (m)</td>
</tr>
</tbody>
</table>
\[ f = \text{force (N) or, alternatively, } \pi W_0^2/\lambda_e (\text{m}) \]
\[ \mathbf{f}_{cf} = \text{centrifugal-force vector (N)} \]
\[ F = \text{GW flux (W/m}^2 \text{)} \]
\[ G = \text{universal gravitational constant} = 6.67423\times10^{-11} (\text{m}^3/\text{kg-s}^2) \]
\[ h = \text{metric perturbation, the amplitude of the GW as a function of time (meters/meter)} \]
\[ h = \text{Planck’s constant} = 6.626068 \times 10^{-34} (\text{m}^2 \text{kg s}^{-1}) \]
\[ \ddot{h}_y = \frac{\partial h_y}{\partial t} \]
\[ I = \text{moment of inertia (kg-m}^2 \text{)} \]
\[ k = \text{Boltzmann’s constant} = 1.3807\times10^{-23} (\text{JK}^{-1}) \]
\[ k_e = 2\pi/\lambda_e (\text{m}^{-1}) \]
\[ l = \text{distance along z-axis (m)} \]
\[ m = \text{mass of an object (kg)} \]
\[ N_x^{(1)} = \text{the x-component of the perturbative photon flux.} \]
\[ N_c^{(1)} = \text{the detection photon number in the duration } \delta t \]
\[ N_x^{(0)} = \text{the x-component of the background photon flux.} \]
\[ n_x^{(1)} = \text{the perturbative photon flux (PPF) density propagating along the x-axis (s}^{-1}) \]
\[ P = \text{Gaussian beam power (W)} \]
\[ P = \text{magnitude of the power of a gravitational-radiation source (W)} \]
\[ R = z + f/\lambda_e (\text{m}) \]
\[ r = \text{radial distance to an object; alternately, the effective radius of gyration, or } r = (x^2 + y^2)^{1/2} (\text{m}) \]
\[ t = \text{time (s)} \]
\[ W_0 = \text{minimum spot radius (m)} \]
\[ W = W_0 [1 + (z/f)^2]^{1/2} \]
\[ x = \text{x-coordinate or distance to the fractal membrane (m)} \]
\[ y = \text{y-coordinate (m)} \]
\[ y = \text{this y variable vector expresses that background static magnetic field } B \text{ is pointed along the y-axis} \]
\[ y_0 = \text{amplitude of electric (or magnetic) field of the Gaussian beam (Vm}^{-1}) \]
\[ z = \text{z-coordinate sectional axis of the Gaussian beam (m)} \]
\[ \delta s = \text{cross-sectional area of the Gaussian beam (m}^2 \text{)} \]
\[ \delta t = \text{differential time interval (s)} \]
\[ \Delta = \text{arbitrary phase factor} \]
\[ \Delta = \text{small increment} \]
\[ \Delta f_{cf} = \text{incremental of centrifugal force (N)} \]
\[ \Delta f_i = \text{incremental tangential force (N)} \]
\[ \Delta t = \text{time increment (s)} \]
\[ \lambda = \text{wavelength (m)} \]
\[ \mu_0 = \text{the vacuum permittivity} = 4\pi \times 10^{-7} (\text{H/m}) \]
\[ \nu = \text{frequency (s}^{-1}) \]
\[ \psi_o = \text{electrical field of Gaussian beam (Vm}^{-1}) \]
\[ \omega = \text{angular rotational rate (rad/s)} \]

**Subscripts**
1 refers to mass one or length one  
g gravitational wave  
2 refers to mass two or length two  
i, j index 1, 2, 3  
cf centrifugal  
o initial  
e electromagnetic  
r radial  
GB Gaussian beam  
t tangential  
GW gravitational wave  
x, y x-, y-directions

**REFERENCES**

Woodward, J. F. personal communication by E-mail, June 27, 2005.